UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level
Advanced International Certificate of Education

MARK SCHEME for the June 2004 question papers

9709 MATHEMATICS

9709/01	**Paper 1 (Pure 1), maximum raw mark 75**
9709/02	**Paper 2 (Pure 2), maximum raw mark 50**
9709/03, 8719/03	**Paper 3 (Pure 3), maximum raw mark 75**
9709/04	**Paper 4 (Mechanics 1), maximum raw mark 50**
9709/05, 8719/05	**Paper 5 (Mechanics 2), maximum raw mark 50**
9709/06, 0390/06	**Paper 6 (Probability and Statistics 1), maximum raw mark 50**
9709/07, 8719/07	**Paper 7 (Probability and Statistics 2), maximum raw mark 50**

These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Grade thresholds taken for Syllabus 9709 (Mathematics) in the June 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	63	56	31
Component 2	50	37	33	18
Component 3	75	61	55	29
Component 4	50	38	34	18
Component 5	50	36	32	17
Component 6	50	38	34	19
Component 7	50	42	37	22

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √"marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/01

MATHEMATICS
Paper 1 (Pure 1)

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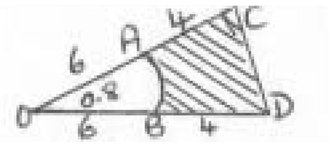
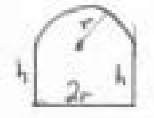
1. (i) a/(1–r) = 256 and a = 64	M1	Use of correct formula
→ r = ¾	A1	Correct only
	[2]	
(ii) S_{10} = 64(1–0.75^{10}) (1–0.75)	M1	Use of correct formula – 0.75^{10} not 0.75^9
→ S_{10} = 242	A1	Correct only
	[2]	
2. $\int_0^1 \sqrt{3x+1}dx = (3x+1)^{1.5} \div 1.5$	B1	Ml for $(3x+1)^{1.5} \div 1.5$
then 3	M1	For division by 3
→ [ ] at 1 – [ ] at 0	M1	Must attempt [ ] at x=0 (not assume it is 0) and be using an integrated function
→ 16/9 – 2/9 = 14/9 or 1.56	A1	Fraction or decimal. (1.56+C loses this A1)
	[4]	
3. (i) $\sin^2\theta + 3\sin\theta\cos\theta = 4\cos^2\theta$		
divides by $\cos^2\theta$	M1	Knowing to divide by $\cos^2\theta$
→ $\tan^2\theta + 3\tan\theta = 4$	A1	Correct quadratic (not nec = 0)
	[2]	
(ii) Solution tan θ = 1 or tan θ = –4	M1	Correct solution of quadratic = 0
→ θ = 45° or 104.0°	A1 A1	Correct only for each one.
	[3]	
4. (i) Coeff of x^3 = 6C3 x 2^3	B1 B1	B1 for 6C3 B1 for 2^3
=160	B1	B1 for 160
	[3]	
(ii) Term in x^2 = 6C2 x 2^2 = 60	B1	B1 for 60 (could be given in (i))
reqd coeff = 1 x (i) – 3 x 60	M1	Needs to consider 2 terms
→ –20	A1	co
	[3]	
5. 		
(i) Area of sector = ½ 6^2 0.8 (14.4)	M1	Use of ½$r^2\theta$ with radians
Area of triangle = ½.10^2.sin0.8 (35.9)	M1	Use of ½absinC or ½ bh with trig
→ Shaded area = 21.5	A1	Correct only
	[3]	
(ii) Arc length = 6 x 0.8 (4.8)	M1	Use of s=rθ with radians
CD (by cos rule) or 2 x 10sin0.4 (7.8)	M1 A1	Any correct method – allow if in (i)
→ Perimeter = 8 + 4.8 + 7.8 = 20.6	A1	Correct only
	[4]	

6. (i) eliminates x (or y) completely	M1	Needs x or y removed completely
→ $x^2+x-6=0$ or $y^2-17y+66=0$	A1	Correct only (no need for = 0)
Solution of quadratic = 0	DM1	Equation must = 0.
→ (2, 6) and (–3, 11)	A1	Everything ok.
	[4]	
(ii) Midpoint = (–½, 8½)	B1 √	For his two points in (i)
Gradient of line = –1		Use of y-step x-step (beware fortuitous)
Gradient of perpendicular = 1	M1	Use of $m_1m_2 = -1$
→ y – 8½ = 1 (x + ½) (or y = x + 9)	M1 A1	Any form – needs the M marks.
	[4]	
7. (i) Differentiate y=18/x → $-18x^{-2}$	M1	Any attempt at differentiation
Gradient of tangent = –½	A1	For –½
Gradient of normal = 2	DM1	Use of $m_1m_2 = -1$
Eqn of normal y–3 = 2(x–6) (y=2x–9)	DM1	Correct method for eqn of line
If y = 0, x = 4½	A1	Ans given – beware fortuitous answers.
	[5]	
(ii) Vol = $\pi\int\frac{324}{x^2}dx=\pi\left[-324x^{-1}\right]$.	M1 A1	Use of $\int y^2dx$ for M. correct(needs π) for A
Uses value at x=6 – value at x= 4.5	DM1	Use of 6 and 4.5
-54π – – 72π = 18π	A1	Beware fortuitous answers (ans given)
	[4]	
8. (i) 2h + 2r + πr = 8	M1	Reasonable attempt at linking 4 lengths + correct formula for ½C or C.
→ h = 4 – r – ½πr 	A1	Co in any form with h subject.
	[2]	
(ii) A=2rh+½πr²→ A = r(8–2r–πr) + ½πr²	M1	Adds rectangle + ½xcircle (eqn on own ok)
→ A = 8r – 2r² – ½πr²	A1	Co beware fortuitous answers (ans given)
	[2]	
(iii) dA/dr = 8 – 4r – πr	M1 A1	Knowing to differentiate + some attempt
= 0 when r = 1.12 (or 8/(4+π))	DM1 A1	Setting his dA/dr to 0. Decimal or exact ok.
	[4]	
(iv) $d^2A/dr^2 = -4-\pi$	M1	Looks at 2nd differential or other valid complete method.
This is negative → Maximum	A1	Correct deduction but needs d^2A/dr^2 correct.
	[2]	

9. $\overrightarrow{OA}=\begin{pmatrix}1\\3\\-1\end{pmatrix},\overrightarrow{OB}=\begin{pmatrix}3\\-1\\3\end{pmatrix},\overrightarrow{OC}=\begin{pmatrix}4\\2\\p\end{pmatrix},\overrightarrow{OD}=\begin{pmatrix}-1\\0\\q\end{pmatrix}$		Condone notation throughout. Allow column vectors or **i**,**j**,**k** throughout
(i) $\overrightarrow{AB}$ = **b**–**a** = 2**i** – 4**j** + 4**k**	M1	Use of **b–a,** rather than **b+a or a–b**
Unit vector = (2**i** – 4**j** + 4**k**) √(2^2+4^2+4^2)	M1	Dividing by the modulus of "his" $\overrightarrow{AB}$
= ± (2**i** –4**j** + 4**k**) 6	A1 [3]	Co (allow – for candidates using **a–b**)
(ii) $\overrightarrow{OA}.\overrightarrow{OC}$ = 4 + 6 – p = 0 for 90° → p = 10	M1 DM1 A1 [3]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Setting to 0 + attempt to solve co
(iii) $(-2)^2 + 3^2 + (q+1)^2 = 7^2$	M1	Correct method for length with **±d–a, d+a**
→ $(q+1)^2 = 36$ or $q^2 + 2q = 35$	A1	Correct quadratic equation
q = 5 and q = –7	DM1 A1 or B1 B1 [4]	Correct method of solution. Both correct. Or B1 for each if $(q+1)^2$=36, q=5 only.
10. f: x ↦ x^2–2x, g: x ↦ 2x+3		
(i) $x^2 - 2x - 15 = 0$ End-points –3 and 5	M1 A1	Equation set to 0 and solved. Correct end-points, however used
→ x < –3 and x > 5	A1 [3]	Co-inequalities – not ≤ or ≥
(ii) Uses dy/dx = 2x–2 = 0 or $(x-1)^2 - 1$ Minimum at x = 1 or correct form	M1 A1	Any valid complete method for x value Correct only
Range of y is f(x) ≥ –1	A1	Correct for his value of "x" – must be ≥
No inverse since not 1 : 1 (or equivalent)	B1 [4]	Any valid statement.
(iii) gf(x) = 2(x^2 – 2x) + 3 (2x^2 – 4x +3)	M1	Must be gf not fg – for unsimplified ans.
b^2 – 4ac = 16 – 24 = –8 → –ve	M1	Used on quadratic=0, even if fg used.
→ No real solutions.	A1 [3]	Must be using gf and correct assumption and statement needed.
[or gf(x)=0 ⟶ f(x)=–3/2. Imposs from (ii)]		
(iv) y = 2x + 3 correct line on diagram	B2,1,0 [2]	3 things needed –B1 if one missing.
Either inverse as mirror image in y=x or y = g^{-1}(x) = ½ (x–3) drawn		• g correct, • g^{-1} correct – not parallel to g • y=x drawn or statement re symmetry

DM1 for quadratic equation. Equation must be set to 0.
Formula ⟶ must be correct and correctly used – allow for numerical errors though in b^2 and –4ac.
Factors ⟶ attempt to find 2 brackets. Each bracket then solved to 0.

June 2004

GCE AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/02

MATHEMATICS
Paper 2 (Pure 2)

UNIVERSITY *of* CAMBRIDGE
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1		Use logarithms to linearise an equation	M1	
		Obtain $\frac{x}{y} = \frac{\ln 5}{\ln 2}$ or equivalent	A1	
		Obtain answer 2.32	A1	**3**
2	**(i)**	Use the given iterative formula correctly at least ONCE with $x_1 = 3$	M1	
		Obtain final answer 3.142	A1	
		Show sufficient iterations to justify its accuracy to 3 d.p.	A1	**3**
	(ii)	State any suitable equation e.g. $x = \frac{1}{5}\left(4x + \frac{306}{x^4}\right)$	B1	
		Derive the given answer α (or x) = $\sqrt[5]{306}$	B1	**2**
3	**(i)**	Substitute x = 3 and equate to zero	M1	
		Obtain answer α = –1	A1	**2**
	(ii)	At any stage, state that x = 3 is a solution	B1	
		EITHER: Attempt division by (x–3) reaching a partial quotient of $2x^2 + kx$	M1	
		Obtain quadratic factor $2x^2 + 5x + 2$	A1	
		Obtain solutions x = –2 and x = -½	A1	
		OR: Obtain solution x = –2 by trial and error	B1	
		Obtain solution x = -½ similarly	B2	**4**
		[If an attempt at the quadratic factor is made by inspection, the M1 is earned if it reaches an unknown factor of $2x^2 + bx + c$ and an equation in b and/or c.]		
4	**(i)**	State answer R = 5	B1	
		Use trigonometric formulae to find α	M1	
		Obtain answer α = 53.13°	A1	**3**
	(ii)	Carry out, or indicate need for, calculation of $\sin^{-1}(4.5/5)$	M1	
		Obtain answer 11.0°	A1√	
		Carry out correct method for the second root e.g. 180° – 64.16° – 53.13°	M1	
		Obtain answer 62.7° and no others in the range	A1√	**4**
		[Ignore answers outside the given range.]		
	(iii)	State least value is 2	B1√	**1**
5	**(i)**	State derivative of the form $(e^{-x} \pm xe^{-x})$. Allow $xe^x \pm e^x$ {via quotient rule}	M1	
		Obtain correct derivative of $e^{\pm x} - xe^{-x}$	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer x = 1	A1	**4**
	(ii)	Show or imply correct ordinates 0, 0.367879…, 0.27067…	B1	
		Use correct formula, or equivalent, with h = 1 and three ordinates	M1	
		Obtain answer 0.50 with no errors seen	A1	**3**
	(iii)	Justify statement that the rule gives an under-estimate	B1	**1**

6	**(i)**	State that $\frac{dx}{dt}=2+\frac{1}{t}$ or $\frac{dy}{dt}=1-\frac{4}{t^2}$, or equivalent	B1	
		Use $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	M1	
		Obtain the given answer	A1	**3**
	(ii)	Substitute t = 1 in $\frac{dy}{dx}$ and both parametric equations	M1	
		Obtain $\frac{dy}{dx}=-1$ and coordinates (2, 5)	A1	
		State equation of tangent in any correct horizontal form e.g. x + y = 7	A1√	**3**
	(iii)	Equate $\frac{dy}{dx}$ to zero and solve for t	M1	
		Obtain answer t = 2	A1	
		Obtain answer y = 4	A1	
		Show by any method (but <u>not</u> via $\frac{d}{dt}(y')$) that this is a minimum point	A1	**4**
7	**(i)**	Make relevant use of the cos(A + B) formula	M1*	
		Make relevant use of cos2A and sin2A formulae	M1*	
		Obtain a correct expression in terms of cosA and sinA	A1	
		Use $\sin^2 A = 1 - \cos^2 A$ to obtain an expression in terms of cosA	M1(dep*)	
		Obtain given answer correctly	A1	**5**
	(ii)	Replace integrand by $\frac{1}{4}\cos 3x + \frac{3}{4}\cos x$, or equivalent	B1	
		Integrate, obtaining $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x$, or equivalent	B1 + B1√	
		Use limits correctly	M1	
		Obtain given anser	A1	**5**

June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

MATHEMATICS AND HIGHER MATHEMATICS
Paper 3 (Pure 3)

UNIVERSITY *of* CAMBRIDGE
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1		Show correct sketch for $0 \le x < \frac{1}{2}\pi$	B1	
		Show correct sketch for $\frac{1}{2}\pi < x < \frac{3}{2}\pi$ or $\frac{3}{2}\pi < x \le 2\pi$	B1	
		Show completely correct sketch	B1	**3**
		[SR: for a graph with $y = 0$ when $x = 0$, π, 2π but otherwise of correct shape, award B1.]		
2	*EITHER*:	State or imply non-modular inequality $(2x+1)^2 < x^2$ or corresponding quadratic equation or pair of linear equations $(2x + 1) = \pm x$	B1	
		Expand and make a reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1	
		Obtain critical values $x = -1$ and $x = -\frac{1}{3}$ only	A1	
		State answer $-1 < x < -\frac{1}{3}$	A1	
	OR:	Obtain the critical value $x = -1$ from a graphical method , or by inspection, or by solving a linear inequality or equation	B1	
		Obtain the critical value $x = -\frac{1}{3}$ (deduct B1 from B3 if extra values are obtained)	B2	
		State answer $-1 < x < -\frac{1}{3}$	B1	**4**
		[Condone $\le$ for $<$; accept –0.33 for $-\frac{1}{3}$.]		
3	*EITHER*:	State $6y\frac{\mathrm{d}y}{\mathrm{d}x}$ as the derivative of $3y^2$	B1	
		State $\pm 4x\frac{\mathrm{d}y}{\mathrm{d}x} \pm 4y$ as the derivative of $-4xy$	B1	
		Equate attempted derivative of LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	
		Obtain answer 2	A1	
		[The M1 is conditional on at least one of the B marks being obtained. Allow any combination of signs for the second B1.]		
	OR:	Obtain a correct expression for y in terms of x	B1	
		Differentiate using chain rule	M1	
		Obtain derivative in any correct form	A1	
		Substitute $x = 2$ and obtain answer 2 only	A1	**4**
		[The M1 is conditional on a reasonable attempt at solving the quadratic in y being made.]		

4	(i)	State or imply $2^{-x} = \frac{1}{y}$	B1	
		Obtain 3-term quadratic e.g. $y^2 - y - 1 = 0$	B1	2
	(ii)	Solve a 3-term quadratic, obtaining 1 or 2 roots	M1	
		Obtain answer $y = (1 + \sqrt{5})/2$, or equivalent	A1	
		Carry out correct method for solving an equation of the form $2^x = a$, where *a* > 0, reaching a ratio of logarithms	M1	
		Obtain answer *x* = 0.694 only	A1	4
5	(i)	Make relevant use of formula for sin 2θ or cos 2θ	M1	
		Make relevant use of formula for cos 4θ	M1	
		Complete proof of the given result	A1	3
	(ii)	Integrate and obtain $\frac{1}{8}(\theta - \frac{1}{4}\sin 4\theta)$ or equivalent	B1	
		Use limits correctly with an integral of the form $a\theta + b\sin 4\theta$, where $ab \neq 0$	M1	
		Obtain answer $\frac{1}{8}(\frac{1}{3}\pi + \frac{\sqrt{3}}{8})$, or exact equivalent	A1	3
6		Separate variables and attempt to integrate	M1	
		Obtain terms $\frac{1}{3}\ln(y^3 + 1)$ and *x*, or equivalent	A1 + A1	
		Evaluate a constant or use limits *x* = 0, *y* = 1 with a solution containing terms $k\ln(y^3 + 1)$ and *x*, or equivalent	M1	
		Obtain any correct form of solution e.g. $\frac{1}{3}\ln(y^3 + 1) = x + \frac{1}{3}\ln 2$	A1√	
		Rearrange and obtain $y = (2\,e^{3x} - 1)^{\frac{1}{3}}$,or equivalent	A1	6
		[f.t. is on $k \neq 0$.]		
7	(i)	Evaluate cubic when *x* = –1 and *x* = 0	M1	
		Justify given statement correctly	A1	2
		[If calculations are not given but justification uses correct statements about signs, award B1.]		
	(ii)	State $x = \frac{2x^3 - 1}{3x^2 + 1}$, or equivalent	B1	
		Rearrange this in the form $x^3 + x + 1 = 0$ (or *vice versa*)	B1	2

	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer –0.68	A1	
		Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (–0.685, –0.675)	A1	**3**
8	**(i)**	*EITHER*: Solve the quadratic and use $\sqrt{-1} = \mathrm{i}$	M1	
		Obtain roots $\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}$ and $\frac{1}{2} - \mathrm{i}\frac{\sqrt{3}}{2}$ or equivalent	A1	
		OR: Substitute *x* + i*y* and solve for *x* or *y*	M1	
		Obtain correct roots	A1	**2**
	(ii)	State that the modulus of each root is equal to 1	B1√	
		State that the arguments are $\frac{1}{3}\pi$ and $-\frac{1}{3}\pi$ respectively	B1√ + B1√	**3**
		[Accept degrees and $\frac{5}{3}\pi$ instead of $-\frac{1}{3}\pi$. Accept a modulus in the form $\sqrt{\frac{p}{q}}$ or $\sqrt{n}$, where *p*, *q*, *n* are integers. An answer which only gives roots in modulus-argument form earns B1 for both the implied moduli and B1 for both the implied arguments.]		
	(iii)	*EITHER*: Verify $z^3 = -1$ for each root	B1 + B1	
		OR: State $z^3 + 1 = (z+1)(z^2 - z + 1)$	B1	
		Justify the given statement	B1	
		OR: Obtain $z^3 = z^2 - z$	B1	
		Justify the given statement	B1	**2**
9	**(i)**	State or imply f(*x*) $\equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1}$	B1	
		EITHER: Use any relevant method to obtain a constant	M1	
		Obtain one of the values: *A* = –1, *B* = 4 and *C* = –2	A1	
		Obtain the remaining two values	A1	
		OR: Obtain one value by inspection	B1	
		State a second value	B1	
		State the third value	B1	**4**
		[Apply the same scheme to the form $\frac{A}{x-2} + \frac{Bx+C}{x^2-1}$ which has *A* = 4, *B* = –3 and *C* = 1.]		

(ii) Use correct method to obtain the first two terms of the expansion of $(x-1)^{-1}$ or $(x-2)^{-1}$ or $(x+1)^{-1}$ M1

Obtain any correct unsimplified expansion of the partial fractions up to the terms in x^3

(deduct A1 for each incorrect expansion) A1√ + A1√ + A1√

Obtain the given answer correctly A1 **5**

[Binomial coefficients involving –1, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$, are not sufficient for the M1 mark. The f.t. is on *A*, *B*, *C*.]

[Apply a similar scheme to the alternative form of fractions in (**i**), awarding M1*A1√A1√ for the expansions, M1(dep*) for multiplying by *Bx* + *C*, and A1 for obtaining the given answer correctly.]

[In the case of an attempt to expand $(x^2+7x-6)(x-1)^{-1}(x-2)^{-1}(x+1)^{-1}$, give M1A1A1A1 for the expansions and A1 for multiplying out and obtaining the given answer correctly.]

[Allow attempts to multiply out $(x-1)(x-2)(x+1)(-3+2x-\frac{3}{2}x^2+\frac{11}{4}x^3)$, giving B1 for reduction to a product of two expressions correct up to their terms in x^3, M1 for attempting to multiply out at least as far as terms in x^2, A1 for a correct expansion up to terms in x^3, and A1 for correctly obtaining the answer x^2+7x-6 and also showing there is no term in x^3.]

[Allow the use of Maclaurin, giving M1A1√ for f(0) = –3 and f ′(0) = 2, A1√ for f ″(0) = –3, A1√ for f ‴(0) = $\frac{33}{2}$, and A1 for obtaining the given answer correctly (f.t. is on *A*, *B*,*C* if used).]

10 (**i**) State *x*-coordinate of *A* is 1 B1 **1**

(**ii**) Use product or quotient rule M1

Obtain derivative in any correct form e.g. $-\dfrac{2\ln x}{x^3}+\dfrac{1}{x}.\dfrac{1}{x^2}$ A1

Equate derivative to zero and solve for ln *x* M1

Obtain *x* = $e^{\frac{1}{2}}$ or equivalent (accept 1.65) A1

Obtain *y* = $\dfrac{1}{2e}$ or exact equivalent not involving ln A1 **5**

[SR: if the quotient rule is misused, with a 'reversed' numerator or x^2 instead of x^4 in the denominator, award M0A0 but allow the following M1A1A1.]

(**iii**) Attempt integration by parts, going the correct way M1

Obtain $-\dfrac{\ln x}{x}+\displaystyle\int\frac{1}{x}.\frac{1}{x}\,dx$ or equivalent A1

Obtain indefinite integral $-\dfrac{\ln x}{x}-\dfrac{1}{x}$ A1

Use *x*-coordinate of *A* and e as limits, having integrated twice M1

Obtain exact answer $1-\dfrac{2}{e}$, or equivalent A1 **5**

[If *u* = ln *x* is used, apply an analogous scheme to the result of the substitution.]

11 (**i**) *EITHER*: Obtain a vector in the plane e.g. $\overrightarrow{PQ} = -3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ B1

Use scalar product to obtain a relevant equation in a, b, c e.g. $-3a + 4b + c = 0$ or $6a - 2b + c = 0$ or $3a + 2b + 2c = 0$ M1

State two correct equations in a, b, c A1

Solve simultaneous equations to obtain one ratio e.g. $a : b$ M1

Obtain $a : b : c = 2 : 3 : -6$ or equivalent A1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1

[The second M1 is also given if say c is given an arbitrary value and a or b is found. The following A1 is then given for finding the correct values of a and b.]

OR: Substitute for P, Q, R in equation of plane and state 3 equations in a, b, c, d B1

Eliminate one unknown, e.g. d, entirely M1

Obtain 2 equations in 3 unknowns A1

Solve to obtain one ratio e.g. $a : b$ M1

Obtain $a : b : c = 2 : 3 : -6$ or equivalent A1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1

[The first M1 is also given if say d is given an arbitrary value and two equations in two unknowns, e.g. a and b, are obtained. The following A1 is for two correct equations. Solving to obtain one unknown earns the second M1 and the following A1 is for finding the correct values of a and b.]

OR: Obtain a vector in the plane e.g. $\overrightarrow{QR} = 6\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ B1

Find a second vector in the plane and form correctly a 2-parameter equation for the plane M1

Obtain equation in any correct form e.g. $\mathbf{r} = \lambda(-3\mathbf{i} + 4\mathbf{j} + \mathbf{k}) + \mu(6\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + \mathbf{i} - \mathbf{k}$ A1

State 3 equations in x, y, z, λ, and μ A1

Eliminate λ and μ M1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1

OR: Obtain a vector in the plane e.g. $\overrightarrow{PR} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ B1

Obtain a second vector in the plane and calculate the vector product of the two vectors, e.g. $(-3\mathbf{i} + 4\mathbf{j} + \mathbf{k})\times(3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ M1

Obtain 2 correct components of the product A1

Obtain correct product e.g. $6\mathbf{i} + 9\mathbf{j} - 18\mathbf{k}$ or equivalent A1

Substitute in $2x + 3y - 6z = d$ and find d or equivalent M1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1 **6**

(**ii**)	*EITHER*:	State equation of *SN* is $\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$ or equivalent	B1√	
		Express *x*, *y*, *z* in terms of λ e.g. $(3 + 2\lambda, 5 + 3\lambda, -6 - 6\lambda)$	B1√	
		Substitute in the equation of the plane and solve for λ	M1	
		Obtain $\overrightarrow{ON} = \mathbf{i} + 2\mathbf{j}$, or equivalent	A1	
		Carry out method for finding *SN*	M1	
		Show that *SN* = 7 correctly	A1	
	OR:	Letting $\overrightarrow{ON} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, obtain two equations in *x*, *y*, *z* by equating scalar product of $\overrightarrow{NS}$ with two of $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RP}$ to zero	B1√+ B1√	
		Using the plane equation as third equation, solve for *x*, *y*, and *z*	M1	
		Obtain $\overrightarrow{ON} = \mathbf{i} + 2\mathbf{j}$, or equivalent	A1	
		Carry out method for finding *SN*	M1	
		Show that *SN* = 7 correctly	A1	
	OR:	Use Cartesian formula or scalar product of $\overrightarrow{PS}$ with a normal vector to find *SN*	M1	
		Obtain *SN* = 7	A1	
		State a unit normal $\hat{\mathbf{n}}$ to the plane	B1√	
		Use $\overrightarrow{ON} = \overrightarrow{OS} \pm 7\hat{\mathbf{n}}$	M1	
		Obtain an unsimplified expression e.g. $3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k} \pm 7(\frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k})$	A1√	
		Obtain $\overrightarrow{ON} = \mathbf{i} + 2\mathbf{j}$, or equivalent, only	A1	**6**

June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/04

MATHEMATICS
Paper 4 (Mechanics 1)

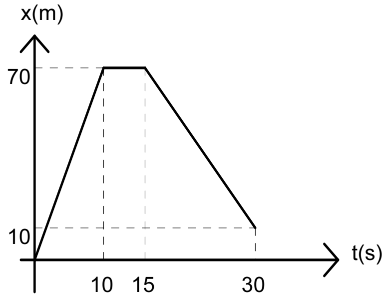
UNIVERSITY *of* CAMBRIDGE
International Examinations

1	(i)	$F = 13\cos\alpha$	M1	For resolving forces horizontally
		Frictional component is 12 N	A1 2	
	(ii)	$R = 1.1\times 10 + 13\sin\alpha$	M1	For resolving forces vertically (3 terms needed)
		Normal component is 16 N	A1 2	
	(iii)	Coefficient of friction is 0.75	B1 ft 1	

2		$X = 100 + 250\cos 70^{\circ}$	B1	
		$Y = 300 – 250\sin 70^{\circ}$	B1	
		$R^2 = 185.5^2 + 65.1^2$	M1	For using $R^2 = X^2 + Y^2$
		$R = 197$	A1 ft	ft only if one B1 is scored or if the expressions for the candidate's X and Y are those of the equilibrant
		$\tan\alpha = 65.1/185.5$	M1	For using $\tan\alpha = Y/X$
		$\alpha = 19.3$	A1 ft 6	ft only if one B1 is scored SR for sin/cos mix (max 4/6) $X = 100 + 250\sin 70^{\circ}$ and $Y = 300 – 250\cos 70^{\circ}$ (334.9 and 214.5) B1 Method marks as scheme M1 M1 $R = 398$ N and $\alpha = 32.6$ A1

OR

		316(.227766..) or 107(.4528..) or 299(.3343..)	B1	Magnitude of the resultant of two of the forces
		71.565 …° or 37.2743 .. ° or –51.7039 .. °	B1	Direction of the resultant of two of the forces
		$R^2 = 316.2^2 + 250^2 – 2\times 316.2\times 250\cos 38.4^{\circ}$ $R^2 = 107.5^2 + 100^2 – 2\times 107.5\times 100\cos 142.7^{\circ}$ $R^2 = 299.3^2 + 300^2 – 2\times 299.3\times 300\cos 38.3^{\circ}$	M1	For using the cosine rule to find R
		$R = 197$	A1 ft	ft only if one B1 is scored
		$\sin(71.6 - \alpha) = 250\sin 38.4 \div 197$ $\sin(37.3 - \alpha) = 100\sin 142.7 \div 197$ $\sin(51.7 + \alpha) = 300\sin 38.3 \div 197$	M1	For using the sine rule to find α
		$\alpha = 19.3^{\circ}$	A1 ft	ft only if one B1 is scored

3	(i)	Distance AC is 70 m	B1	
		$7\times 10 - 4\times 15$	M1	For using |AB| = |AC| - |BC|
		Distance AB is 10 m	A1 3	
	(ii)		M1	Graph consists of 3 connected straight line segments with, in order, positive, zero and negative slopes. $x(t)$ is single valued and the graph contains the origin
			A1	1st line segment appears steeper than the 3rd and the 3rd line segment does not terminate on the t-axis
			A1 ft 3	Values of t (10, 15 and 30) and x (70, 70, 10) shown, or can be read without ambiguity from the scales SR (max 1out of 3 marks) For first 2 segments correct B1

4	**(i)**	KE = 0.2g(0.7) Kinetic energy is 1.4 J	M1 A1	 2	For using KE = PE lost and PE lost = *mgh*
	(ii)	$R = 0.2\times10\times\cos16.3^{\circ}$ F = 0.288 N WD = 0.72 J or a = 1.36 or resultant downward force = 0.272 N KE = 1.4 – 0.72 or KE = ½ 0.2(2 × 1.36 × 2.5) or 0.272 × 2.5 Kinetic energy is 0.68 J	B1 B1 ft B1 ft M1 A1 ft	 5	1.92 From 0.15R (may be implied by subsequent exact value 0.72, 1.36 or 0.68) From 2.5F or from $0.2a = 0.2\times10\times(7/25) - F$ (may be implied by subsequent exact value 0.68) For using KE = PE lost – WD or KE = ½ mv^2 and $v^2 = 2as$ or KE = resultant downward force × 2.5
5	**(i)**	$10t^2 - 0.25t^4$ (+C) Expression is $10t^2 - 0.25t^4$ - 36	M1 DM1 A1	 3	For integrating v For including constant of integration and attempting to evaluate it
	(ii)	Displacement is 60 m	A1 ft	1	Dependent on both M marks in (i); ft if there is not more than one error in $s(t)$
	(iii)	$(t^2 - 36)(1 - 0.25t^2) = 0$ Roots of quadratic are 4, 36 t = 2, 6	M1 A1 A1 ft	 3	For attempting to solve s = 0 (depends on both method marks in (i)) or $\int_0^t v\,dt$ =36 (but not –36) for t^2 by factors or formula method ft only from 3 term quadratic in t^2
6	**(i)**	DF – 400 = 1200 × 0.5 20000 = 1000v Speed is 20 ms^{-1}	M1 A1 M1 A1	 4	For using Newton's 2nd law (3 terms needed) For using $P = Fv$
	(ii)	20000/v - 400 = 0 $v_{\max}$ = 50 ms^{-1}	M1 A1	 2	For using $P = Fv$ and Newton's 2nd law with a = 0 and F = 400 AG
	(iii)	$20000 = \frac{1500000}{\Delta T}$ or distance = 1500 000/400 = 3750 and time = 3750/50 Time taken is 75 s	M1 A1	 2	For using $P = \frac{\Delta W}{\Delta T}$ or for using 'distance = work done/400' and 'time =distance/50'

7	**(i)**	$25 = 30t - 5t^2$ ➔ $t^2 - 6t + 5 = 0$ ➔ $(t - 1)(t - 5) = 0$ or $v^2 = 30^2 - 500$; $t_{up} = (20 - 0)/10$	M1		For using $25 = ut - ½ gt^2$ and attempting to solve for t or for using $v^2 = u^2 - 2g(25)$ and $t_{up} = (v - 0)/g$
		$t = 1, 5$ or $t_{up} = 2$	A1		
		Time = 5 – 1 = 4 s or Time = 2×2 = 4s or $1 < t < 5$	A1	3	
	(ii)	$s_1 = 30t - 5t^2$ and $s_2 = 10t - 5t^2$	M1		For using $s = ut - ½ gt^2$ for P_1 and P_2
		$30t - 10t = 25$	M1		For using $s_1 = s_2 + 25$ and attempting to solve for t
		$t = 1.25$	A1		
		$v_1 = 30 - 10 \times 1.25$ or $v_2 = 10 - 10 \times 1.25$ or $v_1^2 = 30^2 - 2 \times 10(29.6875)$ or $v_2^2 = 10^2 - 2 \times 10(4.6875)$	M1		For using $v = u - gt$ (either case) or for calculating s_1 and substituting into $v_1^2 = 30^2 - 2 \times 10s_1$ or calculating s_2 and substituting into $v_2^2 = 10^2 - 2 \times 10s_2$
		Velocities 17.5ms^{-1} and -2.5ms^{-1}	A1	5	

OR

	(ii)	$v_1 = 30 - 10t$, $v_2 = 10 - 10t$ ➔ $v_1 - v_2 = 20$	M1		For using $v = u - gt$ for P_1 and P_2 and eliminating t
			M1		For using $v^2 = u^2 - 2gs$ for P_1 and P_2 and then $s_1 = s_2 + 25$
		$(30^2 - v_1^2) \div 20 = (10^2 - v_2^2) \div 20 + 25$	A1		
		$v_1 - v_2 = 20$, $v_1^2 - v_2^2 = 300$	M1		For solving simultaneous equations in v_1 and v_2
		Velocities are 17.5 ms^{-1} and -2.5 ms^{-1}	A1	5	
	(iii)	$t_{up} = 3$	B1		
		$3 - 1.25$	M1		For using $t_{\text{up and above}} = t_{up} - t_{\text{equal}}$
		Time is 1.75 s or $1.25 < t < 3$	A1	3	

OR

	(iii)	$0 = 17.5 - 10t$	M2		For using $0 = u - gt$ with u equal to the answer found for v_1 in **(ii)**
		Time is 1.75 s or $1.25 < t < 3$	A1		SR (max 1 out of 3 marks) $0 = 17.5 + 10t$ B1 ft

June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/05, 8719/05

MATHEMATICS AND HIGHER MATHEMATICS
Paper 5 (Mechanics 2)

UNIVERSITY of CAMBRIDGE
International Examinations

Mechanics 2

1		For taking moments about the edge of the platform	M1	
		($75g$ x 0.9 = $25g$ x x + $10g$ x 1.1 (3 term equation)		
		Two terms correct (unsimplified)	A1	
		Completely correct (unsimplified)	A1	
		Distance *MC* = 3.16m	A1	**4**
		<u>NB:</u> If moments taken about other points, the force of the platform on the plank must be present at the edge of the platform for M1		
2	(i)	Evaluates $\frac{2r\sin\alpha}{3\alpha}\times\cos\frac{\pi}{4}$	M1	
		Obtains given answer correctly	A1	**2**
	(ii)	For taking moments about *AB*	M1	
		{(5 x 10 + $\frac{1}{4}\pi$ 5^2)$\bar{x}$ = (5 x 10) x 5 + $\frac{1}{4}\pi$ 5^2(10 + $\frac{20}{3\pi}$)}		
		For the total area correct and the moment of the rectangle correct (unsimplified)	A1	
		For the moment of *CDE* correct (unsimplified)	A1	
		Distance is 7.01 cm	A1	**4**
3		For applying Newton's 2nd law and using $a = v\frac{dv}{dx}$	M1	
		$0.6v\frac{dv}{dx} = -\frac{3}{x^3}$	A1	
		For separating the variables and integrating	M1	
		$0.3v^2 = -\frac{3x^{-2}}{(-2)}$ (+C)	A1 ft	
		(ft omission of minus sign in line 2 only)		
		For using = 0 when x = 10	M1	
		$v^2 = \frac{5}{x^2} - \frac{1}{20}$ (aef)	A1 ft	
		(ft wrong sign in line 4 only)		
		Speed is $\frac{\sqrt{3}}{2}$ ms^{-1} (=0.866)	A1	**7**

4	**(i)**	Distance of the rod from the hinge is $\frac{2.4}{2.5}(0.7)$ or 0.7cos16.26° (=0.672)	B1	
		[May be implied in moment equation]		
		For taking moments about the hinge (3 term equation)	M1	
		0.672*F* = 68 x 1.2 + 750 x 2.4	A1 ft	
		Force is 2800 N	A1	**4**
	(ii)	*X* = 784 (ft for 0.28*F*)	B1 ft	
		For resolving vertically (4 term equation)	M1	
		Y = 1870 (ft for 0.96*F* – 818)	A1 ft	**3**

SR: For use of 680 N for weight of the beam: (i) B1, M1, A0. In (ii) ft 680, so 3/3 possible.

5	**(i)**	For using EPE = $\frac{\lambda x^2}{2L}$	M1	
		EPE gain = $2\left(\frac{200x^2}{2\times 4}\right)$ (=50x^2)	A1	
		GPE loss = 10g (4 + *x*)	B1	
		For using the principle of conservation of energy to form an equation containing EPE, GPE and KE terms	M1	
		[½10 ^{2} + 50x^2 = 10g (4 + *x*)]		
		Given answer obtained correctly	A1	**5**

ALTERNATIVE METHOD:

		T = $\frac{200x}{4}$	B1	
		$100-2\left(\frac{200x}{4}\right)=10v\frac{dv}{dx}$	M1	
		$\frac{1}{2}v^2=10x-5x^2$ (+C)	A1	
		Use *x* = 0, ^{2} = 8g	M1	
		^{2} = 10(8 + 2*x* – x^2)	A1	
	(ii)	For using = 0 and factorizing or using formula method for solving	M1	
		x = 4 (only)	A1	**2**

6 **(i)** 2 = $VT\sin 35° - 5T^2$ or 2 = $25\tan 35° - \frac{25^2 \times 10}{2V^2\cos^2 35°}$ B1

25 = $VT\cos 35°$ B1

For obtaining V^2 or T^2 in $AV^2 = B$ or $CT^2 = D$ form where *A,B,C,D* are numerical M1

[[(25tan35° – 2)$\cos^2 35°]V^2$ = 3125 (aef) or
$5T^2$ = 25tan35° – 2 (aef)]

V = 17.3 or T = 1.76 A1

T = 1.76 or V = 17.3 (ft VT = 30.519365) B1 ft **5**

(ii) For using $\dot{y} = V\sin 35° - gT$ (must be component of *V* for M1) M1

$\dot{y}_M$ (= 9.94 – 17.61 = -7.67) < 0 → moving downwards A1 ft

(ft on *V* and *T*)

For using $_M{}^2 = (V\cos 35°)^2 + \dot{y}_M{}^2$ M1

($_M{}^2 = ((14.20)^2 + (-7.67)^2)$ or

For using the principle of conservation of energy

($\frac{1}{2}m(v_M{}^2 - 17.3^2) = -mg \times 2$)

$_M$ = 16.1 ms^{-1} A1 **4**

LINES 1 AND 2 ALTERNATIVE METHODS

<u>EITHER</u> Compare 25 with $\frac{1}{2}R\left(\frac{1}{2}\frac{v^2\sin 70°}{g}\right)$ M1

25 > 14.1 → moving downwards A1

<u>OR</u> Compare 1.76 with time to greatest height $\left(\frac{V\sin 35°}{g}\right)$ M1

1.76 > 0.994 → moving downwards A1

<u>OR</u> $\frac{dy}{dx} = \tan 35° - \frac{g.10}{V^2\cos^2 35°}(= -0.54)$ used M1

As tanϕ is negative → moving downwards A1

7	(i)	$T\cos 60° = 0.5g$ (T = 10)	B1	
		For applying Newton's 2nd law horizontally and using $a = \frac{v^2}{r}$	M1	
		(must be a component of T for M1)		
		$T\sin 60° = \frac{0.5v^2}{0.15\sin 60°}$ (for an equation in V^2)	A1	
		For substituting for T	M1	
		= 1.5	A1	5
	ALTERNATIVELY:			
		$a = \frac{v^2}{0.15\sin 60°}$	B1	
		For applying Newton's 2nd law perpendicular to the string	M1	
		$0.5g\cos 30° = 0.5(a\cos 60°)$	A1	
		For substituting for a	M1	
		(5cos30° = 0.5 ^{2}/0.15tan60°) (for an equation in V^2)		
		= 1.5	A1	
	(ii)	**(a)** $T\sin 45° = \frac{0.5(0.9)^2}{0.15\sin 45°}$	B1	
		Tension is 5.4 N	B1	2
		(b) For resolving forces vertically	M1	
		$5.4\cos 45° + R = 0.5g$	A1 ft	
		Force is 1.18 N	A1	3

June 2004

GCE A AND AS LEVEL
AICE

MARK SCHEME

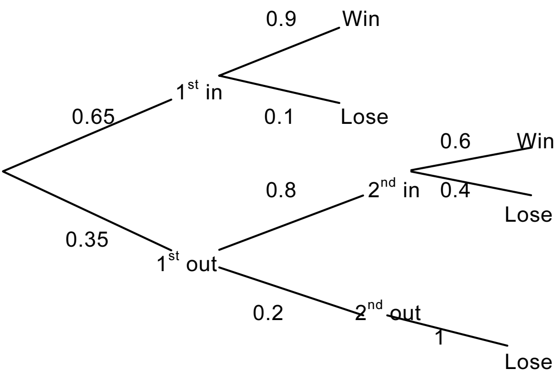
MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/06, 0390/06

MATHEMATICS
Paper 6 (Probability and Statistics 1)

UNIVERSITY *of* CAMBRIDGE
International Examinations

<table>
<tr><td>1 (i) $\bar{x}_A = 139 \quad (138.75)$
$\sigma_A = 83.1$</td><td>B1
B1
2</td><td>For the mean
For the sd</td></tr>
<tr><td>(ii) team B
smaller standard deviation</td><td>B1
B1 dep
2</td><td>Independent mark
Need the idea of spread
SR If team A has a smaller sd then award B1only for ‘teamA, smaller sd’</td></tr>
<tr><td>2 (i) axes and labels
points
(3,0) (15,160) (20,320) (35,480) (60,640)</td><td>B1
B1
B1
3</td><td>For correct uniform scales and labels on both axes, accept Frequency, %CF, Number of people, allow axes reversed, allow halves
For 3 correct points
All points correct and reasonable graph incl straight lines</td></tr>
<tr><td>(ii) accept 60 – 70 for straight lines
40 – 70 for curve</td><td>M1
A1
2</td><td>For subtracting from 640 can be implied
For correct answer, reasonably compatible with graph</td></tr>
<tr><td>3 (i)
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>P($X = x$)</td><td>$^{11}/_{36}$</td><td>$^{9}/_{36}$</td><td>$^{7}/_{36}$</td><td>$^{5}/_{36}$</td><td>$^{3}/_{36}$</td><td>$^{1}/_{36}$</td></tr>
</table></td><td>M1
A1
A1
3</td><td>For 36 in the uncancelled denominator somewhere, accept decimals eg 0.305 recurring or 0.306 etc
For 3 correct probabilities
All correct</td></tr>
<tr><td>(ii) E(X) = $1 \times {}^{11}/_{36} + 2 \times {}^{9}/_{36} + 3 \times {}^{7}/_{36} +$
$4 \times {}^{5}/_{36} + 5 \times {}^{3}/_{36} + 6 \times {}^{1}/_{36} = {}^{91}/_{36}$</td><td>M1
A1
2</td><td>For calculation of $\sum xp$ where all probs < 1</td></tr>
<tr><td>4 (i) $z = \frac{350-450}{120}$
$= -0.833$
% small = 1 – 0.7975 = 0.2025 or 20.25%</td><td>M1
A1
A1
3</td><td>For standardising accept 120 or √120, no cc
For correct z value, + or -, accept 0.83
For answer rounding to 0.202 or 0.203</td></tr>
<tr><td>(ii) 0.7975 ÷ 2 = 0.39875 each
$\Phi z_2 = 0.60125$
$z_2 = 0.257$
$x = 120 \times 0.257 + 450$
= 481</td><td>M1
M1dep
M1
M1dep
A1
5</td><td>For dividing their remainder by 2
For adding their above two probs together or subt from 1
For finding the z corresponding to their probability
For converting to x from a z value
For answer, rounding to 481</td></tr>
</table>

5 (a) (i) $3 \times 5 \times 3 \times 2$ or $_3C_1 \times {}_5C_1 \times {}_3C_1 \times 2$ = 90	M1 A1 **2**	For multiplying $3 \times 5 \times 3$ For correct answer
(ii) $(3 \times 5 \times 2) + (3 \times 3) + (5 \times 2 \times 3)$ = 69	M1 M1 A1 **3**	For summing options that show S&M,S&D,M&D $3 \times 5 \times a + 3 \times 3 \times b + 5 \times 3 \times c$ seen for integers a,b,c For correct answer
(b) $_{14}C_5 \times {}_9C_5 \times {}_4C_4$ or equivalent = 252252	M1 M1 A1 **3**	For using combinations not all $_{14}C$… For multiplying choices for two or three groups For correct answer NB 14!/5!5!4! scores M2 and A1if correct answer
6 (i) 	B1 B1 B1 B1 **4**	For top branches correct (0.65, 0.9, 0.1) For bottom branches correct (0.35, 0.8, 0.2) For win/lose option after 2nd in (0.6, 0.4) For all labels including final lose at end of bottom branch
(ii) $0.65 \times 0.1 + 0.35 \times 0.8 \times 0.4 + 0.35 \times 2$ = 0.247	M1 M1 A1 **3**	For evaluating 1st in and lose seen For 1st out 2nd in lose, or 1st out 2nd out lose For correct answer
(iii) $\dfrac{0.65 \times 0.1}{0.247}$ = 0.263 (= 5/19)	M1 A1ft **2**	For dividing their 1st in and lose by their answer to (ii) For correct answer, ft only on 0.65×0.1/their (ii)

7 **(i)** $P(0) = (0.8)^{15}$ (= 0.03518) $P(1) = {}_{15}C_1 \times (0.2) \times (0.8)^{14}$ (= 0.1319) $P(2) = {}_{15}C_2 \times (0.2)^2 \times (0.8)^{13}$ (= 0.2309) $P(X \le 2) = 0.398$	B1 B1 B1 **3**	For correct numerical expression for P(0) For correct numerical expression for P(1) or P(2) For answer rounding to 0.398
(ii) $1 - (0.8)^n \ge 0.85$ $0.15 \ge (0.8)^n$ n = 9	M1 M1 dep A1 **3**	For an equality/inequality involving 0.8, *n*, 0.85 For solving attempt (could be trial and error or lg) For correct answer
(iii) $\mu = 1600 \times 0.2 = 320$, $\sigma^2 = 1600 \times 0.2 \times 0.8 = 256$ $P(X \ge 290)$ or $P(X<350)$ $= 1 - \Phi\left(\frac{289.5 - 320}{\sqrt{256}}\right) = 1 - \Phi(-1.906)$ $= \Phi(1.906) = 0.972$	B1 M1 M1 M1 A1 **5**	For both mean and variance correct For standardising , with or without cc, must have $\sqrt{\ }$ on denom For use of continuity correction 289.5 or 290.5 For finding an area > 0.5 from their *z* For answer rounding to 0.972

June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/07, 8719/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 (i) H$_0$: μ = 15 or p = 0.25 H$_1$: μ > 15 or p > 0.25	B1 **1**	For H$_0$ and H$_1$ correct
(ii) Test statistic $z = \pm\frac{21.5-15}{\sqrt{60\times0.25\times0.75}}=1.938$	M1	For attempt at standardising with or without cc, must have $\sqrt{\ }$ something with 60 in on the denom
OR test statistic $z = \pm\frac{22/60 - 0.5/60 - 15/60}{\sqrt{\frac{0.25\times0.75}{60}}} = 1.938$	A1	For 1.94 (1.938)
CV z = 1.645	M1	For comparing with 1.645 or 1.96 if 2-tailed, signs consistent, or comparing areas to 5%
In CR Claim justified	A1ft **4**	For correct answer(ft only for correct one-tail test)
2 (i) Mean = 3.5 + 2.9 + 3.1 = 9.5	B1	9.5 as final answer
Var = $0.3^2 + 0.25^2 + 0.35^2$ (=0.275)	M1	For summing three squared deviations
St dev = 0.524	A1 **3**	For correct answer
(ii) $z = \frac{9-9.5}{\sqrt{\frac{their\ \text{var}}{4}}} = -1.907$	M1	For standardising, no cc
or $z = \frac{36-38}{\sqrt{(4 \times their\ \text{var})}} = -1.907$	M1	For $\sqrt{\frac{their\ \text{var}}{4}}$ or √(4 x *their* var) in denom - no 'mixed' methods.
$\Phi(1.907) = 0.9717 = 0.972$	A1 **3**	For correct answer
3 (i) E(2*X*-3*Y*) = 2E(*X*) –3E(*Y*) = 16 – 18	M1	For multiplying by 2 and 3 resp and subt
= - 2	A1 **2**	For correct answer
(ii) Var (2*X*-3*Y*) = 4Var (*X*) +9Var (*Y*)	B1	For use of var (*Y*) = 6
= 19.2 + 54	M1	For squaring 3 and 2
	M1	For adding variances (and nothing else)
= 73.2	A1 **4**	For correct final answer
4 (i) $\bar{x} = 375.3$	B1	For correct mean (3.s.f)
$\sigma^2_{n-1} = 8.29$	M1	For legit method involving *n*-1, can be implied
	A1 **3**	For correct answer
(ii) p = 0.19 or equiv.	B1	For correct p
$0.19 \pm 2.055\times\sqrt{\frac{0.19\times0.81}{200}}$	M1	For correct form $p \pm z\times\sqrt{\frac{pq}{n}}$ either/both sides
	B1	For z = 2.054 or 2.055
$0.133 < p < 0.247$	A1 **4**	For correct answer

5 (i) $\frac{c-54}{3.1/\sqrt{10}}$ = -1.282 c = $54-1.282\times\frac{3.1}{\sqrt{10}}$ = 52.74	B1 M1 A1 A1 **4**	For + or – 1.282 seen For equality/inequality with their *z* (±) (must have used tables), no $\sqrt{10}$ needed (c can be numerical) For correct expression (c can be numerical, but signs must be consistent) For correct GIVEN answer. No errors seen.
(ii) P($\bar{x}$ > 52.74) = 1 - $\Phi\left(\frac{52.74-51.5}{3.1/\sqrt{10}}\right)$ = $1-\Phi(1.265)=1-0.8971$ = 0.103 or 0.102	B1 M1 A1 A1 **4**	For identifying the outcome for a type II error For standardising , no $\sqrt{10}$ needed For ± 1.265 (accept 1.26-1.27) For correct answer
6 (i) P(5) = $e^{-6}\times\frac{6^5}{5!}$ = 0.161	M1 A1 **2**	For an attempted Poisson P(5) calculation, any mean For correct answer
(ii) P(*X* ≥ 2) = 1 –{P(0) + P(1)} = 1 - $e^{-1.6}(1+1.6)$ = 0.475	B1 M1 A1 **3**	For μ = 1.6, evaluated in a Poisson prob For 1 – P(0) – P(1) or 1 – P(0) – P(1) – P(2) For correct answer
(iii) P(1 then 4 $\mid$ 5) = $\frac{\left(e^{-3}\times 3\right)\times\left(e^{-3}\times\frac{3^4}{4!}\right)}{e^{-6}\times\frac{6^5}{5!}}$ = 0.156 or 5/32	M1 M1 A1 **3**	For multiplying P(1) by P(4) any (consistent) mean For dividing by P(5) any mean For correct answer
7 (i) $c\int_0^5 t(25-t^2)\mathrm{d}t$ = 1 $c\left[\frac{25t^2}{2}-\frac{t^4}{4}\right]_0^5$ = 1 $c\left[\frac{625}{2}-\frac{625}{4}\right]$ = 1 $\Rightarrow c=\frac{4}{625}$	M1 A1 A1 **3**	For equating to 1 and a sensible attempt to integrate For correct integration and correct limits For given answer correctly obtained
(ii) $\int_2^4 ct(25-t^2)\mathrm{d}t=\left[\frac{25ct^2}{2}-\frac{ct^4}{4}\right]_2^4$ = $c[136]-c[46]$ = $\frac{72}{125}$ (0.576)	M1* M1*dep A1 **3**	For attempting to integrate f(*t*) between 2 and 4 (or attempt 2 and 4) For subtracting their value when t = 2 from their value when t = 4 For correct answer
(iii) $\int_0^5 ct^2(25-t^2)\mathrm{d}t$ = $\left[\frac{4}{625}\times\frac{25t^3}{3}-\frac{4}{625}\times\frac{t^5}{5}\right]_0^5$ = $\frac{8}{3}$	M1* A1 M1*dep A1 **4**	For attempting to integrate *t*f(*t*), no limits needed For correct integrand can have *c* (or their *c*) For subtracting their value when t=0 from their value when t=5 For correct answer